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Decision analysis on generation capacity of a wind park

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A R T I C L E I N F O

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A B S T R A C T

The investment decision on generation capacity of a wind park is difficult when wind studies or data are neither available nor sufficient to provide adequate information for developing a wind power project. Although new measurement is possible but it is definitely time consuming. To determine the optimum capacity, decision analysis techniques are proposed in this paper to cope with uncertainties arising from wind speed distribution and power–speed characteristics. The wind speed distribution is modeled from the measured data, the Rayleigh distribution, and the Weibull distribution. The power–speed curve of a wind turbine from cut-in speed to rated speed is modeled by using linear, parabolic, cubic, and quadratic characteristics. The optimization model is formulated as a mixed-integer nonlinear programming problem. The constraints are considered as interval bounds so that a set of feasible solutions is obtained. The optimum solution can be determined by using the profit-to-cost and profit-to-area ratios as performance metrics of investment. Decision analysis rules are then applied to overcome the uncertainty problem and to refine the investment plan. The proposed procedure has been tested with the wind power project of the Electricity Generating Authority of Thailand.

Contents

1. Introduction

The utilization of wind energy has been well known for centuries, mainly for irrigation purposes. It is also known that wind energy can be converted to electrical energy using a wind-turbine generator. However, wind power generation was limited because of high investment cost. Over the last decade, the wind power generation has become financially attractive and dramatically growing as a result of a gradual decrease in investment cost and a promotion of green power generation. Besides, price spikes of oil and natural gas cause operation costs of thermal power generation higher and enable renewable power generation, especially wind power, to be more competitive. Although the wind is free and the generation site is quite environmental friendly, but the unpredictable nature of the wind is a major disadvantage and becomes a critical factor on investment decision. In the area of relatively low wind speed or limited meteorological data, electric utility may encounter a difficult decision either in selecting wind power as a generation source or in determining generation capacity of wind power. In Thailand, for instance, wind speed data have almost

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Nomenclature

A_i	swept area of wind turbine type i (m^2)
C_i	capital cost of wind turbine type i (US$)
$C_{p,i}$	power coefficient of wind turbine type i
D_i	rotor diameter of wind turbine type i (m)
E	energy generation per annum of a wind park (kWh/y)
f	probability density function
F	cumulative distribution function
G	generation capacity of a wind park (kW)
H_i	tower height of wind turbine type i (m)
L	available area of a wind park (m^2)
M	available investment fund of a wind park (US$)
N_i	number of wind turbines type i
$O_{i,k}$	operating and maintenance costs of wind turbine type i in year k (US$)
$P_{i,j}$	power output of wind turbine type i at wind speed level j (kW)
$P_{R,i}$	rated power of wind turbine type i (kW)
P_T	turbine power captured from the wind (kW)
r_k	electricity purchasing rate of wind energy in year k (US$/kWh)
T_j	number of hours per year at wind speed level j (h/y)
v	wind speed (m/s)
V_j	discretized wind speed level j (m/s)
V_m	mean wind speed (m/s)
$V_{I,i}$	cut-in wind speed of wind turbine type i (m/s)
$V_{O,i}$	cut-out wind speed of wind turbine type i (m/s)
$V_{R,i}$	rated wind speed of wind turbine type i (m/s)
W_C	space factor between columns of adjacent wind turbines
W_R	space factor between rows of adjacent wind turbines

Greek symbols

Γ	gamma function
α	coefficient of optimism
δ	discount rate (%/y)
ρ	mass density of air (kg/m^2)
σ	standard deviation of wind speed
$\psi_{p,q}$	utility or valuation function of an alternative p given state of nature q

Subscripts

i	index of turbine type $i = \{1, 2, \ldots, I\}$
j	index of discretized wind speed level $j = \{1, 2, \ldots, J\}$
k	index of year $k = \{1, 2, \ldots, K\}$
p	index of alternative
q	index of state of nature

Superscripts

max	maximum value
min	minimum value

Abbreviations

NPV	net present value
P/A	generation profit-to-area ratio (US$/$m^2$)
P/C	generation profit-to-cost ratio

never been recorded. A recent survey from the Department of Alternative Energy Development and Efficiency reported that the average wind speed is less than 7 m/s [1]. Thus, it is very critical for electric utility to integrate wind power capacity into the generation portfolio given relatively low wind speed condition as well as limited wind data.

Verification of wind speed distribution and power–speed characteristics is important not only for the operation monitoring and maintenance of a wind turbine but also for the efficiency assessment of the wind turbine in terms of energy yield. The warranted curve is numerically calculated and calibrated for the specific site. Although the deviations from the warranted curve were within the error margin but the difference in energy production could be considerable. A profound effect of the air density, wind velocity, wind shear, and turbulence intensity in the flow field, especially in very complex terrain sites, can introduce significant uncertainty in power–speed characteristics.

The potential of wind power for electricity generation has been investigated extensively at several locations and in many countries. The general criteria of investment considered weather data and technical specification of wind energy conversion systems based on economic/financial analysis techniques [2]. The investment strategies were developed exclusively for low wind speed area [3]. The power–speed characteristic was assumed to be linear. The optimum solutions (investment plans) were expressed in terms of number and size of wind turbine models. The screening and ranking method was proposed to identify the most attractive investment plan. To account for the unpredictable nature of the wind, various models were studied to find proper investment planning [4,5] and to minimize economic risks [6].

Wind speed and local weather data have played critical role in determining investment decision [7]. Statistical models, by means of probability density functions of wind speed, were presented and compared with meteorological data [8]. The Weibull and Rayleigh models were recognized that they could represent wind speed distribution over several locations properly, including the distribution in the low wind speed area [9,10]. It was found that the Weibull model can be more properly fitted with the monthly wind data than the Rayleigh model [11]. However, the Weibull model might not properly represent the wind speed distribution when historical data were insufficient or the diurnal effect was considered [12].

This work proposes a solution procedure to economically evaluate generation capacity of a wind park. The objective is to maximize the expected generation profit and minimize risk arising from uncertain wind speed distribution and power–speed characteristic. The major constraints are capital spending, installation area, generation capacity, and, annual energy generation of a wind park. Each constraint was represented as an interval bound so that investment resources and outcomes were flexible. The optimization problem was formulated as a mixed-integer nonlinear programming problem. The problem was solved by using DICOPT® solver under the General Algebraic Modeling System (GAMS®) environment [13,14] to determine the optimal number and capacity of wind turbines to be installed at a wind park. Upon solution, multiple investment alternatives may exist so that selection criteria of investment are further required. It is proposed to define the profit-to-cost and profit-to-area ratios as performance metrics to evaluate each investment alternative. Then, uncertainties were introduced into the problem in terms of various wind speed distributions and power–speed characteristics. Decision analysis under uncertainty was applied to identify the most attractive investment plan. The test model is taken from a wind power project at the Phromthep cape, Phuket, Thailand. This project was under consideration by the Electricity generating Authority of Thailand (EGAT).

2. Mathematical model

2.1. Problem formulation

The optimization problem for determining optimum generation capacity of a wind park is formulated as a mixed-integer nonlinear programming problem. The objective function is to maximize the net present value (NPV) of generation profit, which is defined as generation revenue less (a) capital investment cost and (b) operating and maintenance (O&M) costs. For the sake of simplicity, the purchasing rate of wind energy is assumed to be constant for each year. Note that, in this work, the O&M costs are given as a percentage of the capital cost and the cost of land was ignored. There are four constraints being considered in the problem, i.e., investment budget, installation area, generation capacity, and annual energy generation. Each constraint is bounded within a certain range in such a way that feasible solution is ensured. It should be emphasized that the occupied area of wind turbines is computed by comparing two criteria. The first criterion computes a circular area occupied by each wind turbine by considering the hub height as a radius, while the second criterion computes a rectangular area occupied by each wind turbine by defining row and column spaces between adjacent wind turbines. In practice, a larger area would be required when safety factor is included. In this work, the wind speed is discretized so that power output of each wind turbine must also be computed at each wind speed level. The problem is solved for number and size of each wind turbine type. The problem was coded in GAMS® and solved by using DICOPT®.

Maximize

$$\left(\sum_{k=1}^{K}\frac{r_k}{(1+\delta)^k}\sum_{j=1}^{J}\sum_{i=1}^{I}N_iT_jP_{i,j}\right)-\left(\sum_{i=1}^{I}N_iC_i\right)-\left(\frac{(1+\delta)^k-1}{\delta(1+\delta)^k}\sum_{i=1}^{I}N_iO_i\right) \quad (1)$$

Subject to

$$M^{\min}\le\sum_{i=1}^{I}N_iC_i\le M^{\max} \quad (2)$$

$$L^{\min}\le \mathrm{Max}\left\{\sum_{i=1}^{I}\pi N_iH_i^2,\sum_{i=1}^{I}W_CW_RN_iD_i^2\right\}\le L^{\max} \quad (3)$$

$$G^{\min}\le\sum_{i=1}^{I}N_iP_{R,i}\le G^{\max} \quad (4)$$

$$E^{\min}\le\sum_{j=1}^{J}\sum_{i=1}^{I}N_iT_jP_{i,j}\le E^{\max} \quad (5)$$

2.2. Planning uncertainty from wind speed

Several statistical distributions have been employed to represent wind speed data [15–18]. The Weibull distribution is frequently employed to represent probability of wind speed variation [15–18] because of its flexibility to reasonably fit wind speed data measured as hourly average values from different locations and altitudes. Although the Weibull distribution has been widely accepted to give a good approximation of the hourly variation, but the annual variation remains difficult to predict.

The knowledge of actual wind speeds at tower height (30 m and above) is desirable but limited because most wind data were measured around 10 m above the ground [19]. The variation of wind speed and direction with height is known as *wind shear*. A simple expression to take the wind shear effect into account is available but the expression must be determined empirically [19]. Thus, the degree of accuracy is dependent of the historical data of wind speed.

As a result, it is proposed to consider the uncertainty in wind speed arising from the variation of wind speed distribution over the planning horizon. The wind speed distribution is characterized into 3 models. The first model is obtained by fitting the meteorological data at the wind park location. The second model is the Weibull distribution. The third model is the Rayleigh distribution which is relatively fitted with low-speed wind data.

The general form of the Weibull distribution is given [8,11,12,15,20] as follows:

$$F(v)=1-\exp\left[\left(-\frac{v}{c}\right)^k\right] \quad (6)$$

$$f(v)=\frac{k}{c}\left(\frac{v}{c}\right)^{k-1}\exp\left[\left(-\frac{v}{c}\right)^k\right] \quad (7)$$

The probability of observing wind speed v is determined from shape parameter k and scale parameter c. In general, the values of shape parameter range from 1 to 3 and the values of scale parameter are slightly higher than the mean wind speed [15–18,21]. When the shape parameter of the Weibull distribution is 2, it is called the Rayleigh distribution. The shape and scale parameters can be derived from the mean wind speed V_m and the associated standard deviation σ [7–9,12] as follows:

$$V_m=\int_0^\infty v\,f(v)\,\mathrm{d}v\cong\frac{1}{J}\sum_{j=1}^{J}V_jF(V_j) \quad (8)$$

$$\sigma=\sqrt{\int_0^\infty (v-V_m)^2f(v)\,\mathrm{d}v}\cong\sqrt{\frac{1}{J-1}\sum_{j=1}^{J}(V_j-V_m)^2F(V_j)} \quad (9)$$

$$k=\left(\frac{\sigma}{V_m}\right)^{-1.086} \quad (10)$$

$$c=\frac{V_m}{\Gamma(1+(1/k))} \quad (11)$$

Given the mean wind speed of 5.16 m/s and standard deviation of 2.784 calculated from hourly wind data measured at the Phuket wind station in Thailand, the Weibull shape and scale parameter are 1.954 and 5.823, respectively. The probability distribution functions obtained from the 3 models is illustrated in Fig. 1, while

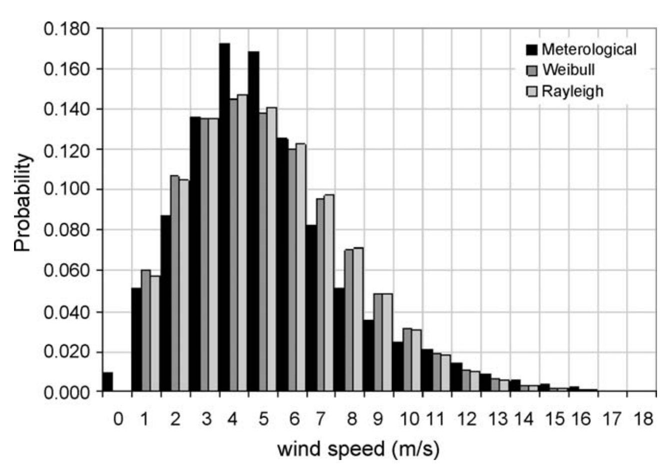


Fig. 1. Comparison of meteorological, Weibull, and Rayleigh distribution models analyzed from hourly wind data at Phuket wind station.

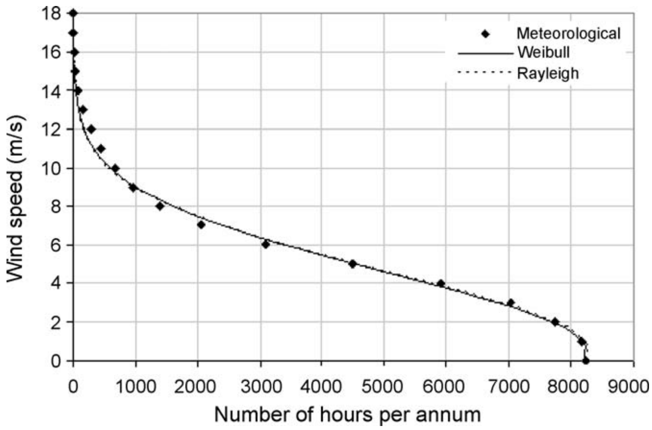


Fig. 2. Comparison of speed duration curves analyzed from hourly wind data at Phuket wind station.

Fig. 2 shows the speed duration curves derived from the 3 models. It can be seen from Fig. 1 that the wind speed mostly varies from 4 to 7 m/s and the maximum wind speed reaches 16 m/s. Thus, the Phuket wind station is in relatively low wind speed region. However, it is noticed from Fig. 2 that more than 70% of the time that the wind speed is higher than 4 m/s so that this location still has potential for wind power generation.

2.3. Planning uncertainty from power–speed curve

The turbine power captured from the wind is the change of kinetic energy over swept area of the turbine and can be expressed as follows:

$$P_T = \tfrac{1}{2}\rho A v^3 C_P \tag{12}$$

The power coefficient C_P depends on the tip speed ratio and the blade pitch angle. The power output from a wind-turbine generator then has a nonlinear relationship with wind speed. The power output starts deliver at cut-in wind speed, reaches its rated value at rated wind speed, and maintains its rated output up to cut-out wind speed. A wind turbine must be shut down when the wind speed is higher than the cut-out wind speed for safety reasons.

Given a variation of wind speed, the power output lies between zero and the rated value most of the time. The relationship between power output of generator and wind speed between the cut-in and rated wind speeds can be expressed in various forms. The first model [7,20,22–25] is given as follows:

$$P_{i,j} = \begin{cases} 0 & 0 \le V_j \le V_{I,i} \\ aV_j^{\phi} - bP_{R,i} & V_{I,i} < V_j < V_{R,i} \\ P_{R,i} & V_{R,i} \le V_j < V_{O,i} \\ 0 & V_{O,i} \le V_j \end{cases} \tag{13}$$

$$a = \frac{P_{R,i}}{V_{R,i}^{\phi} - V_{I,i}^{\phi}} \tag{14}$$

$$b = \frac{V_{I,i}^{\phi}}{V_{R,i}^{\phi} - V_{I,i}^{\phi}} \tag{15}$$

When ϕ is equal to 1, 2, or 3; the model assumes a linear, parabolic, and cubic relationships for the rising curve, respectively. On the other hand, the second model [26,27] assumes a quadratic relationship for the rising curve as follows:

$$P_{i,j} = \begin{cases} 0 & 0 \le V_j \le V_{I,i} \\ \left(a + bV_j + cV_j^2\right)P_{R,i} & V_{I,i} < V_j < V_{R,i} \\ P_{R,i} & V_{R,i} \le V_j < V_{O,i} \\ 0 & V_{O,i} \le V_j \end{cases} \tag{16}$$

$$a = \frac{1}{(V_{I,i} - V_{R,i})^2}\left[V_{I,i}(V_{I,i} + V_{R,i}) - 4V_{I,i}V_{R,i}\left(\frac{V_{I,i} + V_{R,i}}{2V_{R,i}}\right)^3\right] \tag{17}$$

$$b = \frac{1}{(V_{I,i} - V_{R,i})^2}\left[-(3V_{I,i} + V_{R,i}) + 4(V_{I,i} + V_{R,i})\left(\frac{V_{I,i} + V_{R,i}}{2V_{R,i}}\right)^3\right] \tag{18}$$

$$c = \frac{1}{(V_{I,i} - V_{R,i})^2}\left[2 - 4\left(\frac{V_{I,i} + V_{R,i}}{2V_{R,i}}\right)^2\right] \tag{19}$$

Regardless of the rising curve being assumed, the annual energy generation of a wind park can be calculated as follows:

$$E = \sum_{j=1}^{J}\sum_{i=1}^{I} N_i T_j P_{i,j} \tag{20}$$

By assuming that the rated power is 600 kW, the cut-in wind speed is 3 m/s, the rated wind speed is 12.5 m/s, and the cut-out wind speed is 25 m/s; the power output versus wind speed characteristics under different rising curves are shown in Fig. 3. The annual energy generation is then calculated at each wind speed level and shown in Fig. 4. Again, the hourly wind data

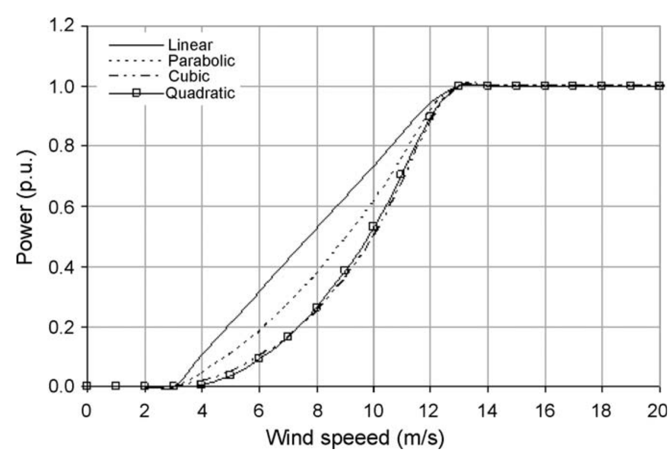


Fig. 3. Comparison of power output versus wind speed characteristics.

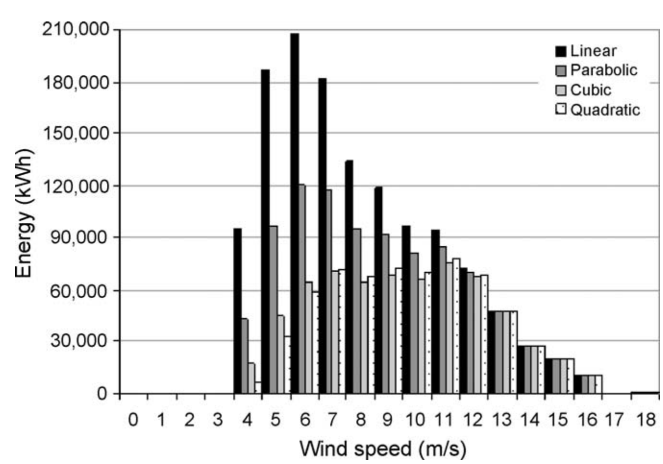


Fig. 4. Comparison of energy generation derived from different power–speed characteristics.

measured at the Phuket wind station are taken. It can be seen that the linear form gives maximum power output, while the quadratic form gives minimum power output. The differences in energy generation are considerable when assuming different rising curves. Therefore, it is proposed to consider the uncertainty in power–speed characteristics as a result of using different rising curves when the wind speed lies between cut-in speed and rated speed. The power–speed characteristics are characterized into 4 forms: linear, parabolic, cubic, and quadratic curves.

3. Decision analysis under uncertainty

Decision analysis is a collection of principles and methods for making decision under different alternatives (possible decisions can be made). There are two main problems involved with decision-making process: uncertainty and risk. Decision-making under uncertainty arises when states of nature of each alternative do not dispose of any probabilistic information on the occurrence of these states. In contrary, decision-making under risk is provided with probabilities on the occurrence of states of nature of each alternative.

In this work, there are five decision principles under uncertainty [28–30] being applied to determine the optimum generation capacity of a wind park. The first decision principle is known as the *maximin* principle. Each alternative is evaluated for the worst possible outcome and the optimum decision is to select the alternative for which the worst outcome is the largest. The second decision principle is known as the *maximax* principle, which is opposite to the first principle. Each alternative is evaluated for the best possible outcome and the optimum decision is to select the alternative for which the best outcome is the largest. The first principle has been considered as a pessimistic decision, while the second principle has been considered as an optimistic decision. The third decision principle, called the *Hurwicz* principle, is defined to reflect a decision that is somewhere between those two extremes. The fourth decision principle is the *Laplace* principle or the principle of *insufficient reason* [29]. Without proper information of the states of nature, all states are then assumed to have equal probability. The optimum solution is to select the alternative for which the average outcome is largest. The fifth principle is called the *savage* principle or the principle of *minimax regret*. The regret value (or opportunity loss) of each state of nature is defined as the difference between the best possible outcome among all alternatives and the outcome of that state. The maximum regret value of each alternative is then identified. The optimum solution is to select the alternative for which the maximum regret is the smallest.

In mathematical form, the utility or valuation ψ of an alternative p given state of nature q can be evaluated by using those five decision principles as follows:

Maximin principle $$\max_p\left[\min_q \quad \psi_{p,q}\right] \tag{21}$$

Maximax principle $$\max_p\left[\max_q \quad \psi_{p,q}\right] \tag{22}$$

Hurwicz principle
$$\max_p\left\{\alpha\left[\max_q \quad \psi_{p,q}\right]+(1-\alpha)\left[\min_q \quad \psi_{p,q}\right]\right\}, \quad 0\le\alpha\le 1 \tag{23}$$

Laplace principle $$\max_p\left[\underset{q}{avg} \quad \psi_{p,q}\right] \tag{24}$$

Savage principle $$\min_p\left[\max_q\left(\max_p \quad \psi_{p,q}\right)-\psi_{p,q}\right] \tag{25}$$

4. Solution methodology

The solution method for determining the optimum capacity of a wind park is described in this section. Firstly, wind speed data at a wind park are analyzed in order to obtain a wind speed distribution. Fig. 5 shows the monthly wind speed data analyzed from the hourly wind data measured at the Phuket wind station. The data were measured at a height of 36 m during the year 2005–2006. It is found that the mean wind speed ranges from 3.5 to 7.6 m/s and the annual mean wind speed is 5.16 m/s which is relatively low. The meteorological distribution model fitted from hourly wind speed data, the Weibull and Rayleigh distribution models calculated from the mean wind speed and associated standard deviation are then determined. Note that the shape and scale parameters of the Weibull model were found to be 1.954 and 5.823, respectively.

Secondly, a set of wind turbines is chosen for installation at a specific site. The important characteristics of a wind turbine are costs, physical dimension, and power–speed characteristics. Table 1 shows the specifications of 10 wind turbines considered for installation at the Phromthep cape, Phuket, Thailand. All wind turbines have relatively low cut-in and rated wind speeds in accordance with the wind speed distribution at the site. The economic life of all wind turbines is assumed to be 20 years. The annual operation and maintenance costs are assumed to be in the range of 1–3% of the capital cost. The power–speed characteristics of each wind turbine are then computed. The rising curve from cut-in speed to rated speed is considered as linear, parabolic, cubic, or quadratic function.

Next, the optimization problem formulated in (1)–(5) is solved for determining number of each wind turbine type. The generation capacity of a wind park is computed by summing up number and capacity of each wind turbine type. The critical input parameters of the problem are available area and investment fund of a wind park. Both parameters are varied within certain ranges so that the investment constraints are flexible. As a consequence, multiple investment plans may be expected upon optimality. Table 2 shows 6 simulation cases assigned for solving the investment problem at the Phromthep cape. On one hand, the available area was bounded up to 100,000 m^2 while the available fund was varied from 1.5 to 2 million US$ (each incremental step was 0.1 million US$). On the other hand, the available area was varied from 30,000 to 60,000 m^2 (each incremental step was 10,000 m^2) while the available fund was bounded up to 4 million US$.

Then, the generation profit-to-cost (P/C) and the generation profit-to-area (P/A) ratios are computed to measure economic

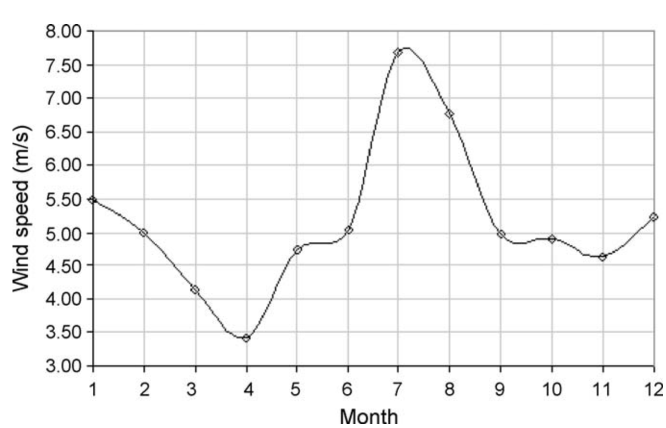


Fig. 5. Monthly wind speed data analyzed from hourly wind data at Phuket wind station.

Table 1
Specification of 10 wind turbines considered for installation at the Phromthep cape.

Turbine type	Rated power (kW)	Height (m)	Diameter (m)	Cut-in speed (m/s)	Rated speed (m/s)	Cut-out speed (m/s)	Capital cost (US$)
W1	1	30	2.5	2.6	11.0	13.0	5000
W2	10	48	7.0	3.6	16.5	19.0	40,000
W3	50	25	15.0	4.6	11.3	22.4	115,000
W4	100	35	21.0	2.5	13.0	25.0	240,000
W5	150	26	20.5	4.0	17.0	25.0	300,000
W6	200	30	30.0	4.0	14.0	25.0	400,000
W7	250	42	29.5	2.5	15.0	25.0	500,000
W8	500	39	39.0	4.0	14.0	25.0	800,000
W9	600	50	43.0	3.0	10.8	20.0	960,000
W10	750	48	43.0	3.0	15.0	25.0	1,000,000

Table 2
Six simulation cases for a wind park at the Phromthep cape.

Case	Available fund (million US$)	Available area (m^2)	Wind speed distribution model
1	1.5–2.0	100,000	Meteorological
2	4.0	30,000–60,000	Meteorological
3	1.5–2.0	100,000	Weibull
4	4.0	30,000–60,000	Weibull
5	1.5–2.0	100,000	Rayleigh
6	4.0	30,000–60,000	Rayleigh

efficiency of each investment plan. The P/C ratio is defined as the ratio of annual generation profit to annual generation cost of a wind park. The P/A ratio is defined as the ratio of the net present value of generation profit to the occupied area of a wind park.

Finally, decision analysis is applied to determine the most attractive investment plan based on the P/C and P/A ratios calculated earlier. The uncertainty sources in this work are wind speed distribution models and power–speed characteristics. The Hurwicz, Laplace, and savage principles are applied to solve the investment problem under uncertainty and their results are then evaluated.

5. Simulation results

The generation capacity of a wind park at the Phromthep cape was planned for 1000 kW. A number of simulations were performed in accordance to 6 cases described in Table 2. Each solution is collected as an investment plan in terms of type and number of wind turbines, investment cost, and occupied area. It is shown in Table 3 that there were 8 investment plans available upon solution. The investment costs of all plans were between 1.5 and 1.92 million US$. The occupied areas were varied from 40,789 to 54,735 m^2.

Tables 4–6 illustrate the optimal investment plans given that the wind speed distribution is based on the meteorological,

Table 3
Comparison of investment plans for a 1000-kW wind park.

Investment plan	Combination of wind turbines	Investment cost (million US$)	Wind park area (m^2)
1	50 kW + 100 kW + 250 kW + 600 kW	1.815	50,779
2	4 × 100 kW + 600 kW	1.920	54,195
3	8 × 50 kW + 600 kW	1.880	54,735
4	4 × 50 kW + 2 × 100 kW + 600 kW	1.900	54,465
5	150 kW + 250 kW + 600 kW	1.760	47,092
6	2 × 200 kW + 600 kW	1.760	54,735
7	50 kW + 2 × 100 kW + 750 kW	1.595	44,340
8	250 kW + 750 kW	1.500	40,789

Table 4
Optimum solutions for a 1000-kW wind park based on meteorological model.

		Power–speed characteristic											
		Linear			Parabolic			Cubic			Quadratic		
		Plan	PC	PA	Plan	PC	PA	Plan	PC	PA	Plan	PC	PA
Investment fund (million US$)	4.0	2	0.424	63.03	2	−0.248	33.28	3	−0.615	16.50	3	−0.560	18.87
	2.0	2	0.424	63.03	2	−0.248	33.28	3	−0.615	16.50	3	−0.560	18.87
	1.9	1	0.437	64.17	4	−0.254	32.50	3	−0.615	16.50	3	−0.560	18.87
	1.8	5	0.356	63.29	6	−0.298	28.22	6	−0.708	11.73	6	−0.691	12.43
	1.7	7	0.180	53.03	7	−0.620	17.06	–	–	–	–	–	–
	1.6	7	0.180	53.03	7	−0.620	17.06	–	–	–	–	–	–
	1.5	8	0.246	57.26	8	−0.641	16.50	–	–	–	–	–	–
Area (m^2)	100,000	2	0.424	63.03	2	−0.248	33.28	3	−0.615	16.50	3	−0.560	18.87
	60,000	2	0.424	63.03	2	−0.248	33.28	3	−0.615	16.50	3	−0.560	18.87
	55,000	2	0.424	63.03	2	−0.248	33.28	3	−0.615	16.50	3	−0.560	18.87

Table 5
Optimum solutions for a 1000-kW wind park based on Weibull model.

		Power–speed characteristic											
		Linear			Parabolic			Cubic			Quadratic		
		Plan	PC	PA	Plan	PC	PA	Plan	PC	PA	Plan	PC	PA
Investment fund (million US$)	4.0	2	0.471	65.09	2	−0.207	35.10	3	−0.573	18.33	3	−0.502	21.38
	2.0	2	0.471	65.09	2	−0.207	35.10	3	−0.573	18.33	3	−0.502	21.38
	1.9	1	0.488	66.45	4	−0.201	34.81	3	−0.573	18.33	3	−0.502	21.38
	1.8	5	0.406	65.67	6	−0.251	30.09	6	−0.684	12.68	6	−0.655	13.84
	1.7	7	0.208	54.28	7	−0.614	17.33	–	–	–	–	–	–
	1.6	7	0.208	54.28	7	−0.614	17.33	–	–	–	–	–	–
	1.5	8	0.268	58.27	8	−0.642	16.45	–	–	–	–	–	–
Area (m^2)	100,000	2	0.471	65.09	2	−0.207	35.10	3	−0.573	18.33	3	−0.502	21.38
	60,000	2	0.471	65.09	2	−0.207	35.10	3	−0.573	18.33	3	−0.502	21.38
	55,000	2	0.471	65.09	2	−0.207	35.10	3	−0.573	18.33	3	−0.502	21.38

Table 6
Optimum solutions for a 1000-kW wind park based on Rayleigh model.

		Power–speed characteristic											
		Linear			Parabolic			Cubic			Quadratic		
		Plan	PC	PA	Plan	PC	PA	Plan	PC	PA	Plan	PC	PA
Investment fund (million US$)	4.0	2	0.464	64.80	2	−0.223	34.37	3	−0.598	17.24	3	−0.527	20.31
	2.0	2	0.464	64.80	2	−0.223	34.37	3	−0.598	17.24	3	−0.527	20.31
	1.9	1	0.480	66.10	4	−0.219	34.02	3	−0.598	17.24	3	−0.527	20.31
	1.8	5	0.398	65.28	6	−0.270	29.33	6	−0.708	11.72	6	−0.608	12.86
	1.7	7	0.197	53.78	7	−0.633	16.49	–	–	–	–	–	–
	1.6	7	0.197	53.78	7	−0.633	16.49	–	–	–	–	–	–
	1.5	8	0.257	57.76	8	−0.661	15.60	–	–	–	–	–	–
Area (m^2)	100,000	2	0.464	64.80	2	−0.223	34.37	3	−0.598	17.24	3	−0.527	20.31
	60,000	2	0.464	64.80	2	−0.223	34.37	3	−0.598	17.24	3	−0.527	20.31
	55,000	2	0.464	64.80	2	−0.223	34.37	3	−0.598	17.24	3	−0.527	20.31

Weibull, and Rayleigh models, respectively. In each model, the power–speed characteristics are represented by linear, parabolic, cubic, and quadratic functions. The investment plans were sorted by the investment fund available for a wind park. The results from the 3 distribution models are similar so that it can be concluded that all models are properly fitted. But, the results are significantly changed when the power–speed relationship is in different forms.

It is mentioned from the results that the P/C and P/A ratios were directly proportional to the investment fund. Both ratios are higher (lower) when the investment fund is greater (smaller). When the power–speed curve is in either linear or parabolic form, there were 6 investment plans emerged as optimal solutions. The investment plan numbers 2, 7, and 8 were obtained as the solutions of both forms. The investment plan numbers 1 and 5 were obtained as the solutions of only the linear form, while investment plan numbers 4 and 6 were obtained as the solutions of only the parabolic form. When the power–speed curve is in either cubic or quadratic form, the optimal solutions reduced to 2 investment plans (plan numbers 3 and 6) and the solutions were not feasible when the investment fund is insufficient. This can be explained by the fact that the energy generation obtained from either the cubic or quadratic form is less than the energy generation obtained from either the linear or parabolic form. As a result, a wind park would require more investment fund to seek for feasible solution and a set of optimal solutions is smaller, given that the energy generation is limited.

When the power–speed curve is not in linear form, the P/C ratios were negative so that the investment is not profitable. This is a direct result of limited energy generation of a wind park when the power–speed curve is not linear. Note that the purchasing electricity rates from small power producers were assumed in the simulations so that a wind park may not be competitive when compared with other power producers. A financial subsidy such as feed-in tariff or production tax credit may be required to financially justify the investment.

The uncertainty is then included in the investment problem of a wind park. All 8 investment plans become the alternatives for investment. The analysis is divided into 3 stages. The first stage considers only the uncertainty in wind speed distribution models. The second stage considers only the uncertainty in power–speed curves. The third stage considers the uncertainties from both sources. Table 7 summarizes the analysis results after applying various decision principles. In all three stages, the investment plan numbers 1 and 3 were the most attractive alternatives. In the first stage, the investment plan number 1 was dominant regardless of the wind distribution model and then became the most attractive alternative. In the second and third stages; the investment plan number 1 was the most attractive alternative when applying the

Table 7
Optimum investment plans for a 1000-kW wind park after applying decision analysis.

Decision principle	Source of uncertainty		
	Wind Speed distribution	Power–speed characteristic	Both
Hurwicz (α = 0) or maximin	1	3	3
Hurwicz (α = 0.25)	1	3	3
Hurwicz (α = 0.50)	1	1	1
Hurwicz (α = 0.75)	1	1	1
Hurwicz (α = 1) or maximax	1	1	1
Laplace	1	3	3
Savage	1	3	3

Hurwicz principle, given that the values of the index of optimism were 0.50, 0.75, and 1. The investment plan number 3 was the most attractive alternative when applying the Hurwicz principle, given that the values of the index of optimism were 0 and 0.25; as well as when applying the Laplace and savage principles. Thus, it is implied that the investment plan number 1 is justified under an optimistic viewpoint while the investment plan number 3 is justified under a pessimistic viewpoint. Moreover, this implication is consistent with the simulation results in Tables 4–6. The investment plan number 1 was optimal when the power–speed curve is in linear form. Alternatively, the investment plan number 3 was optimal when the power–speed curve is in either cubic or quadratic form. In summary, given that the generation capacity of a wind park at the Phromthep cape is 1000 kW, the optimum investment plan is to install 8 of 50-kW wind turbines and a 600-kW wind turbine. The investment cost is 1.88 million US$ and the wind park requires an area of 54,735 m^2.

6. Conclusion

This work proposes a solution procedure to determine the optimum generation capacity of a wind park. The proposed procedure has been implemented with an investment plan for a 1000-kW wind park at the Phromthep cape, Phuket, Thailand. The optimization model was formulated as a mixed-integer nonlinear programming problem. The investment fund and available area, which are the constraints of the problem, were specified as intervals. Therefore, a number of simulations were solved and multiple solutions were presented upon optimality. Each optimal solution was considered as an investment alternative. The profit-to-cost and profit-to-area ratios were proposed to evaluate each investment alternative. When the investment fund is limited, the investment plans with larger size wind turbines were found to be more profitable than the investment plans with smaller size wind turbines. Meanwhile, a number of investment alternatives were reduced when the investment funded is limited and the power–speed curve is not in linear form. When the power–speed curve is not in linear form, it is also found that the investment of a wind park may not be profitable if either the energy generation is limited or the electricity purchasing rate is too low.

The uncertainties in wind speed distribution models and power–speed characteristics were introduced into the problem. Various decision analysis principles were then applied to cope with such uncertainties. The simulation results indicated that the uncertainty in power–speed characteristics has significant impact on the investment decision. The uncertainty in wind speed distribution models has little impact on the investment decision because the statistical models were properly fitted with the meteorological data.

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